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ELECTRICAL ANALOGIES AND ELECTRONIC COMPUTERS: SURGE AND WATER HAMMER PROBLEMS

By Henry M. Paynter, J. M. ASCE

HYDRAULICS DIVISION

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PAPERS

ELECTRICAL ANALOGIES AND ELECTRONIC COMPUTERS: SURGE AND WATER HAMMER PROBLEMS

By Henry M. Paynter,[1] J. M. ASCE

Synopsis

In the course of extensive investigations in the fields of speed and pressure regulation at the Massachusetts Institute of Technology (M.I.T.), at Cambridge, use has been made of certain electrical-fluid analogies. Also, as an aid to obtaining direct solutions while retaining the basically nonlinear features in the components of the systems being studied, an electronic computer has been found useful.

This paper describes the foundation for these techniques, the types of analogies, methods developed, and equipment employed, together with a few representative results and conclusions drawn from these studies. The particular cases of pressure transients in a uniform pipe and surges in a simple tank have been selected for discussion. However, no such restrictions are inherent in the methods developed.

Emphasis has been placed on the utilization of analog techniques to extend, refine, and clarify analytical procedures to secure a more thorough understanding of the basic hydraulic phenomena and to furnish results that are generally useful and accessible. Brief mention is made of some improved analytical procedures developed in connection with these studies.

Introduction

The interconnection of hydraulic and steam power plants within modern electric power networks gives rise to numerous complex problems concerning both the influence of load fluctuations and frequency control equipment on the stable operation of the generating units, and, conversely, the effects of the transient behavior of the hydraulic and mechanical components of the units

Note.—Written comments are invited for publication; the last discussion should be submitted by **February 1, 1953.**

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on the performance of the electrical network. A representative hydroelectric system is shown schematically in Fig. 1, in which B is the gate opening, H is the head, P is the power, N is the speed, and Q is the discharge. As subscripts, c denotes "conduit," l denotes "transmission line," r denotes "reservoir," and t denotes "surge tank." In the course of this paper, use will be made of dimen-

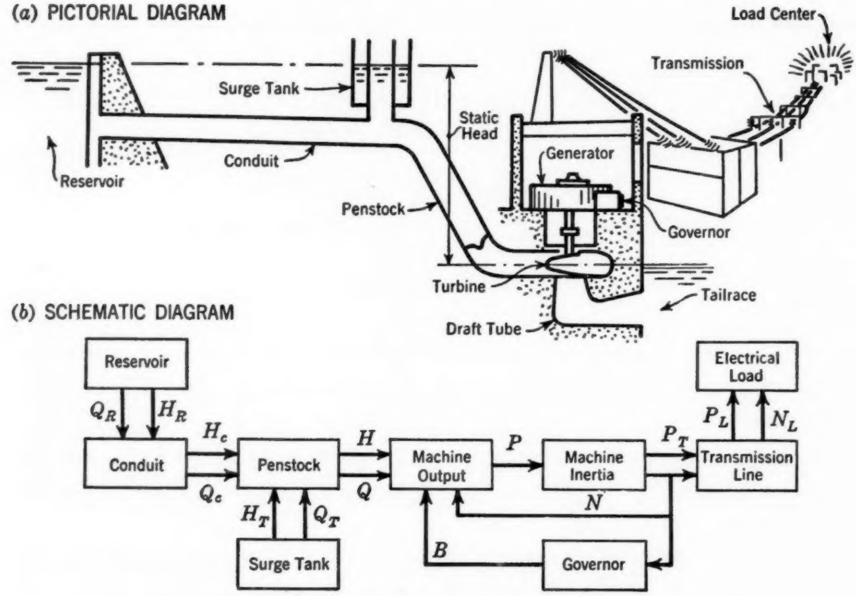

FIG. 1.—TYPICAL HYDROELECTRIC INSTALLATION

sionless incremental variables—the "per-unit" variables familiar to electrical engineers. These will generally be denoted by the lower case letters corresponding to the physical variables; thus, $b = \dfrac{\Delta B}{B_0}$, $h = \dfrac{\Delta H}{H_0}$, $p = \dfrac{\Delta P}{P_0}$, $n = \dfrac{\Delta N}{N_0}$, $q = \dfrac{\Delta Q}{Q_0}$, in which the numerators in each case represent changes from a reference condition and the denominators correspond to this reference or index condition, denoted by the subscript zero.

Most earlier investigations in the regulation field were not extended much beyond the point at which immediate and practical questions were answered. Thus, the classical researches into water hammer and surge phenomena that formed the subject of many pioneering investigations[2,3,4,5,6,7] remained adequate

[2] "Über den hydraulischen Stoss in Wasserleitungsrohren," by N. Joukowsky, *Memoires de l'Academie du Sciences de St. Petersbourg*, Vol. 9, 8th Serie, 1898.

[3] "Théorie générale du mouvement varié de l'eau dans les tuyaux de conduite," by L. Allievi, *Revue de Mecanique*, Janvier et Mars, 1904.

[4] "Teoria del colpo d'ariete," by L. Allievi, *Reale Academia dei Lincei*, Anno CCCIX, 1912.

[5] "The Surge Tank in Water Power Plants," by R. D. Johnson, *Transactions*, ASME, Vol. 30, 1908, p. 443.

[6] "Pressures in Penstocks Caused by Gradual Closing of Turbine Gates," by N. R. Gibson, *Transactions*, ASCE, Vol. 83, 1920, p. 707.

[7] "Sur Théorie des Wasserschlosses bei selbstatig geregelten Turbinenanlagen," by D. Thoma, Olde n burg, Munchen, Germany, 1910.

as long as the various generating stations were loosely tied and operated in relative isolation. However, with the growth of interconnection to the level at which nearly all the plants in a particular system and even entire systems are coupled together, these earlier methods of analysis and special solutions have become inadequate.

Straightforward extension and generalization of the pioneer studies are hampered by the presence of inescapable nonlinearities in the various physical components of the problem. These nonlinearities occur in the hydraulic elements of a hydroelectric plant, for example, in the friction loss in the conduit and penstock, the flow through the wicket gates and tank orifices, and the turbine power and efficiency characteristics. Basically, all physical systems are nonlinear and all analyses arising through linearization procedures are but approximate, agreeing with fact only for those cases in which the deviations resulting from nonlinearity are insignificant; in the present context such is usually the case for equilibrium stability determinations and for small changes in operating conditions.[8,9,10,11,12,13,14,15]

However, there are many instances in which linear treatments are not sufficiently accurate, including, for the hydraulic system, surges and pressure swings for large changes in demand flow. Problems such as these require graphical or numerical solutions, in which techniques are used similar to those employed for water hammer studies[4,6,16,17,18] or surge tank studies.[5,19,20,21] However, to these basic methods must be annexed expressions that reflect the actual plant and load characteristics as well. The tediousness and consumption of time inherent in these methods emphasize the need for more effective techniques for solution, without disregarding essential nonlinearities.

As a first step toward developing new techniques and refined solutions in this field, both for the purpose of extending basic knowledge and to answer immediate questions arising in practice, the M.I.T. Hydrodynamics Laboratory began, in 1947, an investigation into the basic dynamic phenomena involved in the transient performance of power system prime movers, which necessarily

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[8] "Contribution à l'étude des régulateurs de vitesse. Considerations sur le problème de la stabilité," by D. Gaden, Editions La Concorde, Lausanne, Switzerland, 1945.

[9] "Étude de la stabilité d'un réglage automatique de vitesse par des diagrammes vectoriels," by D. Gaden, *Informations Techniques Charmilles No. 2*, Geneve, Switzerland, 1946.

[10] "Influence de certaines caractéristiques intervenant dans la condition de stabilité. À propos du réglage automatique de vitesse des turbines hydrauliques," by D. Gaden, Editions La Concorde, Lausanne, Switzerland, 1949.

[11] "Influence de l'inertie de l'eau sur la stabilité d'un groupe hydroelectrique," by P. Almeras, *La Houille Blanche*, Novembre, 1945.

[12] "Influence des phénomènes de coup de pélier sur le réglage de la vitesse des turbines hydrauliques," by M. Cuenod, *ibid.*, Mars-Avril, 1949.

[13] "La regolazione delle turbine idrauliche," by G. Evangelisti, Zanichelli, Bologna, 1947.

[14] "Sulla stabilità di regolazione nelle installazioni idroelettriche," by G. Evangelisti, *L'Energia Elettrica*, 1946.

[15] "Sulla validità della regola di Thoma per le vasche di oscillazione degli impianti idroelettrici," by E. Scimemi, *ibid.*, 1947.

[16] "Methode graphique générale de calcul des propagations d'ondes planes," by L. Bergeron, *Memoires de la Societé des Ingenieurs Civils de France*, 1937.

[17] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps," by R. W. Angus, *Proceedings*, Inst. of Mech. Eng., Vol. 136, 1937, p. 245.

[18] "Druckstösse in Pumpensteigleitungen," by O. Schnyder, *Schweizerische Bauzeitung*, Vol. 94, Nos. 22 and 23, 1929.

[19] "Théorie de chambres d'équilibre," by J. Calame and D. Gaden, Gautier-Villars, Paris, France, and La Concorde, Lausanne, Switzerland, 1926.

[20] "The Surge Chamber in Hydroelectric Installations," by R. S. Cole, *Selected Engineering Papers, No. 55*, Inst. C. E., London, England, 1927.

[21] "Zur Berechnung von Wassershlössern," by E. Braun, *Schweizerische Bauzeitung*, Band 86, 1925.

required research into the field of hydraulic transients. Certain of these studies are outlined here as well as in a related paper by the author.[22]

Analogs and Computers

One fruitful approach toward understanding these problems has been found in the use of electrical analogs and computers, by means of which solutions may be obtained rapidly and in immediately useful form. These devices fall naturally into two classes,[23,24,25] analogs and computers.

Analogs.—In analogs the prototype system, in effect, is duplicated by a model system whose equivalent components behave according to laws analogous to those governing the prototype.

Computers.—In computers the basic algebraic and differential equations are solved either by (*a*) Digital computation (numerical calculation by discrete steps) or (*b*) analog computation (involving calculation by continuous variation of analogous variables).

Comparisons.—Generally, then, the establishment of a formal analogy between the basic equations describing the behavior of two different physical systems permits the use of previously determined solutions for new cases. Indeed, this is one of the major benefits derived from mathematical analysis itself.

As problems become more complex, with more variables and system parameters, the utility of analog techniques becomes more marked. In particular, most mechanical (or dynamical) problems can be represented by suitable electrical analogs. Since electrical terminology and concepts have become highly developed in the fields of unsteady and periodic motion, these analogies will usually prove fruitful. For a detailed introduction to the basic structure of analog techniques the reader is referred to the literature.[26,27,28,29,30]

With respect to computers, some form of high-speed computer is useful if the system under study involves a large number of variables or many different solutions for varying parameters and conditions. Even linear analyses became awkward for these cases. Computers are also helpful if the system possesses one or more significant nonlinear features that render analytic solutions impossible.

Many problems in the field of speed and pressure regulation possess both of these attributes. In this paper the latter aspect has been emphasized, especially since a number of novel results have been obtained.

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[22] "Methods and Results from M.I.T. Studies in Unsteady Flow," by H. M. Paynter, *Journal*, Boston Soc. of Civ. Engrs., VXXXIX, No. 2, April, 1952.

[23] "High Speed Computing Devices," Eng. Research Associates, McGraw-Hill Book Co., Inc., New York, N. Y., 1950.

[24] "Calculating Instruments and Machines," by D. R. Hartree, Univ. of Illinois Press, Urbana, Ill., 1949.

[25] "Theory of Mathematical Machines," by F. J. Murray, Kings Crown Press, New York, N. Y., 1947.

[26] "Transients in Linear Systems," by M. F. Gardner and J. L. Barnes, John Wiley & Sons, Inc., New York, N. Y., 1942.

[27] "Applied Mathematics for Engineers and Physicists," by L. A. Pipes, McGraw-Hill Book Co., Inc., New York, N. Y., 1946, Chapter VIII.

[28] "Mathematical Methods in Engineering," by T. von Kármán and M. Biot, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, Chapter VI.

[29] "Dynamical Analogies," by H. F. Olson, D. Van Nostrand, Inc., New York, N. Y., 1943.

[30] "Similitude in Engineering," by Glenn Murphy, Ronald Press, New York, N. Y., 1950.

Some Physical Concepts

All continuous devices to transport energy over distances possess many features in common. Such mechanisms may be mechanical shafts, electrical transmission lines, pressure pipe lines, or open channels, all of which can be conceived of as a series of inertial elements linked together by elastic or flexible couplings capable of storing potential energy. In other words, these systems may be visualized in terms of a simple dynamical model consisting of mass cars

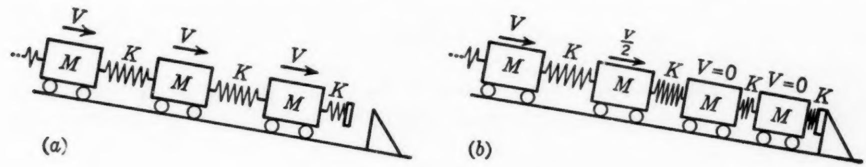


Fig. 2.—Mass-Spring Model

M and coupling springs K, as shown in Fig. 2. The energy terms associated with these elements are

$$\mathrm{KE} = \tfrac{1}{2} M\, V^2 \qquad (1a)$$

and

$$\mathrm{PE} = \tfrac{1}{2} K\, X^2 \qquad (1b)$$

in which KE is kinetic energy and PE is potential energy when the cars are moving with a velocity V and the springs are compressed a distance X.

If the leading car is suddenly stopped, the springs must each in turn absorb the kinetic energy of the adjacent car, and convert it into potential energy. Furthermore, a definite time delay occurs at each unit, in order to decelerate the mass and build up the compression. These two features may be obtained from the conservation of energy and momentum principles as follows:

Energy—

$$\mathrm{KE} = \mathrm{PE} \qquad (2a)$$

$$\tfrac{1}{2} M\, V^2_{\max} = \tfrac{1}{2} K\, X^2_{\max} \qquad (2b)$$

or

$$X_{\max} = \sqrt{\frac{M}{K}}\, V_{\max} = Z_0\, V_{\max} \qquad (3)$$

in which Z_0 = surge impedance = $\sqrt{\dfrac{M}{K}}$.

Momentum—

$$\mathrm{M}\, \Delta V = F\, \Delta T \text{ gives } M\, V_{\max} \sim K\, X_{\max}\, \Delta T \qquad (4)$$

From which

$$\text{delay } \Delta T \sim \frac{M\, V_{\max}}{K\, X_{\max}} = \frac{M}{Z_0\, K} = \sqrt{\frac{M}{K}} \qquad (5)$$

and

$$\frac{c \sim 1}{\Delta T} \sim \sqrt{\frac{K}{M}} \qquad (6)$$

in which c is the propagation velocity. In the case of elastic pressure waves, the corresponding values for the surge impedance Z_0 and propagation velocity c become

$$Z_0 = \frac{c}{g} = \frac{\Delta H}{\Delta V} \qquad (7)$$

in which g is the acceleration due to gravity, and for unbounded fluid,

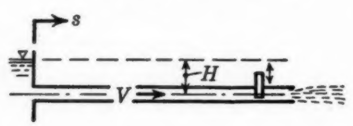
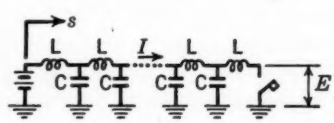
$$c = \sqrt{\frac{E_b}{\rho}} \qquad (8)$$

in which ρ is the density, with Z_0 relating the change in head to the change in velocity. These concepts prove generally useful for all types of transmission problems.

Electrical Analogy

It is demonstrated in Table 1 that a uniform, frictionless pipe line is formally analogous to a uniform, dissipationless electrical transmission line. Thus the

TABLE 1.—Electrical-Hydraulic Analogy

Line	(a) Hydraulic System: Uniform frictionless pipe line	(b) Electrical System: Uniform lossless transmission line
		
1	Inertia equation: $-\frac{\partial V}{\partial s} = \frac{1}{g}\frac{\partial H}{\partial t}$	Voltage drop: $-\frac{\partial E}{\partial s} = L\frac{\partial I}{\partial t}$
2	Continuity equation: $-\frac{\partial V}{\partial s} = \frac{w}{E_b}\left(1 + \frac{E_b}{E_y}\frac{D}{e}\right)\frac{\partial H}{\partial t}$	Line charging: $-\frac{\partial I}{\partial s} = C\frac{\partial E}{\partial t}$
3	Wave equations: $\begin{cases} \frac{\partial^2 H}{\partial t^2} = c^2\frac{\partial^2 H}{\partial s^2} \\ \frac{\partial^2 V}{\partial t^2} = c^2\frac{\partial^2 V}{\partial s^2} \end{cases}$	Wave equations: $\begin{cases} \frac{\partial^2 E}{\partial t^2} = c^2\frac{\partial^2 E}{\partial s^2} \\ \frac{\partial^2 I}{\partial t^2} = c^2\frac{\partial^2 I}{\partial s^2} \end{cases}$
5	Propagation velocity: $c = \sqrt{\frac{E_b/\rho}{1 + \frac{E_b}{E_y}\frac{D}{e}}}$	Propagation velocity: $c = \sqrt{\frac{1}{LC}}$
6	Surge impedance: $Z_0 = \frac{c}{g}$	Surge impedance: $Z_0 = \sqrt{\frac{L}{C}}$
	Reflections:	Reflections:
7	Open end: Pressure node: $\Delta H = 0$	Grounded end: Voltage node: $\Delta E = 0$
8	Reflection factor: $r = -1$	Reflection factor: $r = -1$
9	Closed end: Velocity: $\Delta V = 0$	Open end: Current node: $\Delta I = 0$
10	Reflection factor: $r = +1$	Reflection factor: $r = +1$

ANALOGY

Head $H \longleftrightarrow$ Voltage E
Velocity $V \longleftrightarrow$ Current I

water hammer waves and surges of the hydraulic engineer became the travelling waves, electrical surges, and switching transients of the electrical engineer. The fruits of this analogy are (1) the ability to make use of the many useful tools and concepts developed by the electrical engineer since 1900 and (2) the realization that, when viewed in this light, hydraulic and electrical engineers have similar problems and a common language.

In short, velocity V is analogous to current I, and head H is analogous to voltage E.

As long as dissipation phenomena are negligible in both systems these analogies are strictly valid. Nevertheless, in the practical case resistance and losses must be taken into account. For the complex forms of pipe networks, even in steady flow, computers are useful. With steady flow one successful method of making friction proportional to an exponential power of the flow is embodied in the pipe flow analyzer.[31] For unsteady flow problems, using electronic computers, it is possible to account for this frictional effect satisfactorily through special components described subsequently.

However, even in the loss-free form, many concepts of electrical engineering have direct application to problems of unsteady flow. For example, the ordinary alternating-current vector diagrams furnish valuable clues to the behavior of pipe lines subjected to alternations of flow and pressure. Moreover, the use of surge impedance and the other generalized circuit constants can be extremely profitable.

Resonance Phenomena

An excellent example of the use of electrical concepts for unsteady flow problems can be shown in connection with the resonance phenomena associated with the rhythmic motion of a gate or valve at the end of a pipe line. In such cases, the resultant pressure fluctuations along the pipe may often be substantially in excess of the maximum at the valve for the full gate stroke, for which the governor or gate mechanism timing is usually specified and adjusted. It will also be of interest to compare the results obtained by these techniques with those obtained by following the classical methods.

The solutions of the electrical wave equations for long alternating-current transmission lines, as derived in any standard work on power transmission,[32,33,34] may be put in the form:

$$\vec{E}_x = \vec{E}_s \cosh(\overrightarrow{\alpha x}) - \vec{I}_s \vec{Z}_0 \sinh(\overrightarrow{\alpha x}) \qquad (9)$$

and

$$\vec{I}_x = \vec{I}_s \cosh(\overrightarrow{\alpha x}) - \left(\frac{\vec{E}_s}{Z_0}\right) \sinh(\overrightarrow{\alpha x}) \qquad (10)$$

in which x is the distance along the transmission line from the sending end; $\vec{E}_x$ is the voltage vector at point x; $\vec{E}_s$ is the voltage vector at the sending end;

[31] "Nonlinear Electrical Analogy for Pipe Networks," by Malcolm S. McIlroy, *Proceedings-Separate No. 139*, ASCE, 1952.

[32] "Principles of Electric Power Transmission," by L. F. Woodruff, John Wiley & Sons, Inc., New York, N. Y., 1938.

[33] "Power System Interconnection," by H. Rissik, Pitman, London, England, 1940.

[34] "Standard Handbook for Electrical Engineers," edited by A. E. Knowlton, McGraw-Hill Book Co., Inc., New York, N. Y., 1941, Sect. 13.

$\vec{I}_x$ is the current vector at point x; $\vec{I}_s$ is the current vector at sending end; $\vec{\alpha}$ is the vector propagation constant; and $\vec{Z}_0$ is the vector surge impedance.

If the line is short-circuited (analogous to open reservoir) at $x = L$, then $E_L = 0$ and the sending end voltage and current are related by the expression,

$$\vec{E}_s = \vec{I}_s \vec{Z}_0 \tanh (\overrightarrow{\alpha L}) \quad (11)$$

Furthermore, if the line resistance is negligible (analogous to the frictionless pipe), the propagation constant $\vec{\alpha}$ is given by

$$\vec{\alpha} = \vec{j} \frac{\omega}{c} \quad (12)$$

in which ω is the angular frequency of transmission; c is the propagation velocity; and $j = \sqrt{-1}$.

By the analogy between flow velocity $\longleftrightarrow$ current, and head $\longleftrightarrow$ voltage, one may rewrite this expression directly in hydraulic terms, to obtain corresponding values for head and velocity at the gate, thus:

$$\vec{h} = -2\,K \vec{v} \vec{j} \tan \left(\frac{\pi}{2} \frac{T_n}{T_0} \right) \quad (13)$$

and

$$\vec{v} = + \frac{\vec{h}}{2\,K} \vec{j} \cot \left(\frac{\pi}{2} \frac{T_n}{T_0} \right) \quad (14)$$

in which $T_n = \dfrac{4\,L}{c}$ equals natural period of pipe; T_0 is the period of gate oscillation; $K = \dfrac{c\,V_0}{2\,g\,H_0}$ equals the Allievi parameter; $h = \dfrac{\Delta H}{H_0}$ equals the relative head change; and $v = \dfrac{\Delta V}{V_0} \left(\text{numerically equal to } \dfrac{\Delta Q}{Q_0} \right)$ equals the relative velocity (or flow) change.

Eqs. 13 and 14 demonstrate that the head variation will always lag behind the flow variation by an angle of 90° and that the amplitude ratio of head relative to flow is given by the expression:

$$\left[\frac{h}{v} \right] = 2\,K \tan \left(\frac{\pi}{2} \frac{T_n}{T_0} \right) \quad (15)$$

For small variations, however, the normalized flow v may also be found from the expression:

$$(1 + v) = (1 + b) \sqrt{1 + h} \quad (16)$$

in which $b = \Delta B / B_0 =$ relative gate opening, or, approximately

$$v = b + (\tfrac{1}{2})\, h \quad (17)$$

Combining Eqs. 15 and 17,

$$\left[\frac{h}{b} \right] = -2 \cos \phi_h \text{ (as at angle } \phi_h) \quad (18)$$

and

$$\tan \phi_h = -\frac{1}{K}\cot\left[\frac{\pi}{2}\frac{T_n}{T_0}\right] \qquad (19)$$

A graphical representation of these expressions is shown in Figs. 3 and 4. These results were first obtained by D. Gaden[8] using interval equations derived by L. Allievi.

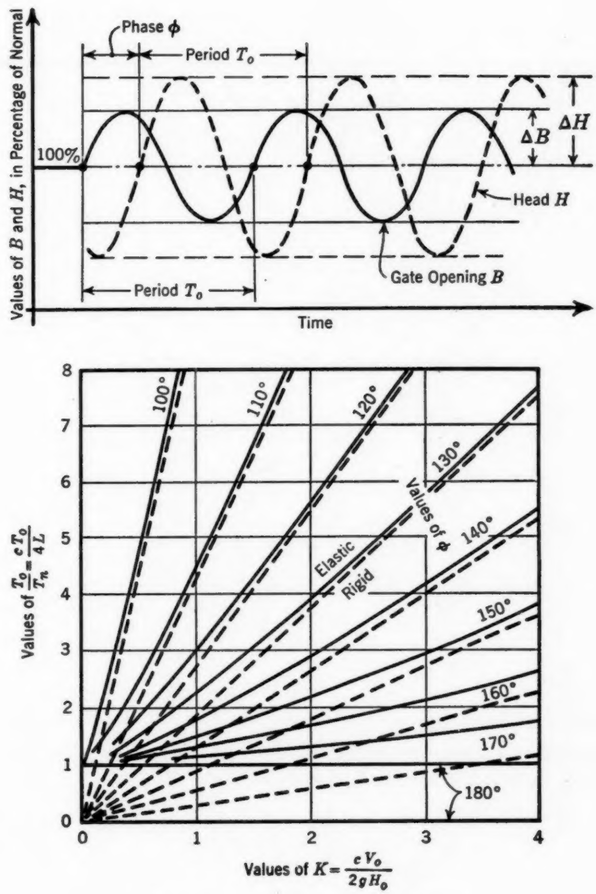


FIG. 3.—WATER HAMMER RESONANCE

For example, in Fig. 4 assume $K = .5$ and $\frac{T_0}{T_n} = 2.5$ to locate the point A; then extend a line OB through point A to point B on the semicircle. The values $\frac{h}{b} = 1.92$ and $\phi_h = 164°$ are read directly. The significance of these curves arises from a consideration of the variations in velocity (or flow) and input torque (or power) caused by oscillations of gate opening. The velocity and

input torque in the normalized (or per-unit) dimensionless form may be expressed by the equations:

Flow—

$$(1 + v) = (1 + b)\ \sqrt{1 + h} \qquad (20)$$

and power—

$$(1 + p) = (1 + h)\ (1 + v) = (1 + b)\ \sqrt{(1 + h)^3} \qquad (21)$$

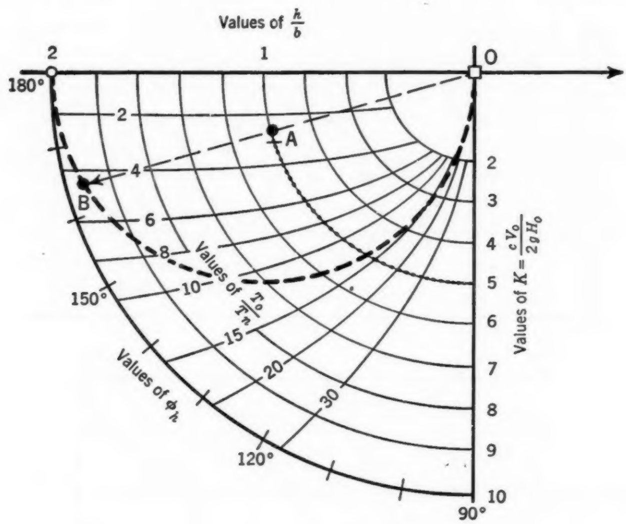


FIG. 4.—RESONANCE DIAGRAM

For small increments Eqs. 20 and 21 become

$$v = b + (\tfrac{1}{2})\ h \qquad (22)$$

and

$$p = b + (\tfrac{3}{2})\ h \qquad (23)$$

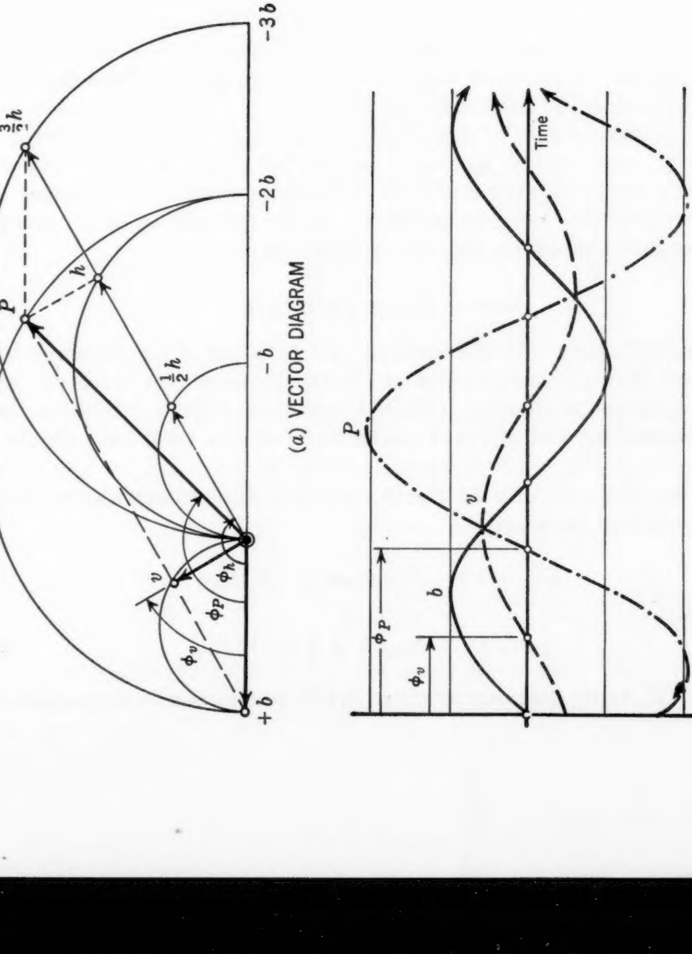
These sinusoidally varying increments can be conceived as rotating vectors (as illustrated in Fig. 5) and the resulting amplitude ratios, making use of the previously determined expressions, become

$$\left[\frac{v}{b}\right] = \sin \phi \qquad (24)$$

and

$$\left[\frac{m}{b}\right] = \sqrt{\tfrac{1}{2}\ (5 + 3 \cos^2 \phi)} \qquad (25)$$

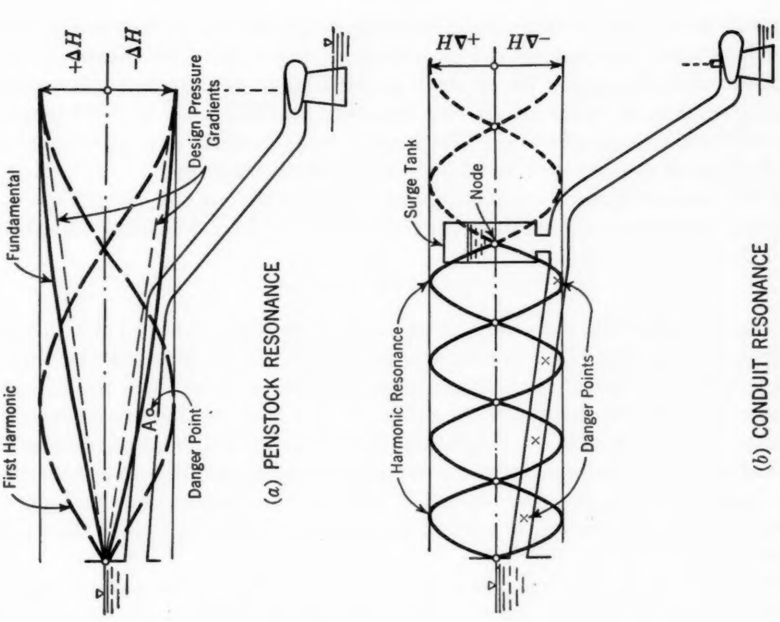
with $\phi = \phi_h$, the lagging angle between the gate vector b and the head vector h, as before, increasing from 90° (for $T_0/T_n \rightarrow \infty$) up to 180° (for $T_0/T_n = 1$). Thus, both the velocity v and the power p always lag behind the gate b, as demonstrated in Fig. 5, in which it should be noted that the tips of the v-vector and p-vector, as well as the multiples of h, all execute circular loci as the frequency increases.



(a) VECTOR DIAGRAM

(b) TIME CURVE

FIG. 5.—RESPONSE CURVES



(a) PENSTOCK RESONANCE

(b) CONDUIT RESONANCE

FIG. 6.—CASES OF RESONANCE

It may be concluded from these analyses that the input power to the prime mover inherently lags behind the gate opening, no matter how slowly the gate is moved, with increasing phase shift ϕ_p for higher frequencies; this circumstance is critical in determining the transient performance of hydroelectric units and clearly indicates one of the fundamental limitations of such units for participation in system load and frequency control programs.

In the Appendix, a numerical example is given showing the use of Figs. 3, 4, and 5 and comparing the results with those obtained by conventional methods.

Application of Results

From the foregoing it is evident that oscillations of the gate or valve produce standing waves in the pipe or systems of pipes with the familiar loops and nodes of all vibratory continuous systems. At the gate end there will usually be a pressure loop, and at the upstream end there is a node at the free surface.

From purely intuitive considerations it is clear that only resonances of the fundamental and odd harmonics can occur under such conditions; but for all of these cases, the full pressure variation will occur at each loop, making the resonance of the harmonics, in general, more dangerous than that of the fundamental.

Moreover, it can be shown that it is possible, under certain conditions, to produce pressure swings along the pipe (through oscillations of the gate that are consistent with the governor timing and stroke) nearly as large as the design values established by the conventional methods. These extreme fluctuations can exist both for the fundamental and the lower harmonics. For example, the pipe line illustrated in Fig. 6(*a*) would be in serious danger of bursting or collapse near point A under the conditions shown.

Another dangerous case of simultaneous resonance can arise between the gate, penstock, and conduit as shown in Fig. 6(*b*). In this case, the surge tank will not serve to trap the elastic waves since a node exists at its base, and, therefore, renders the tank inoperative. Failure resulting from excessive positive or negative pressures may occur at the loops.

Useful Surge Concepts

Considerable insight into the analysis and behavior of simple surge tanks can be gained through consideration of the energy principles involved in the elementary problem of sudden, full-load rejection with a cylindrical tank. The basic dimensions and physical constants of such a tank are indicated in Fig. 7.

For the limiting case of a frictionless conduit, the conservation of energy may be expressed in the form:

$$\text{KE} + \text{PE} = \text{constant} = (\text{KE})_0 \qquad (26)$$

or, specifically,

$$\tfrac{1}{2} M_c V^2 + M_t g h = \tfrac{1}{2} M_c V^2_0 \qquad (27)$$

with M_c and M_t being the mass of water in the conduit and tank, respectively.

With the data of Fig. 7 there results

$$\frac{1}{2}\left(\frac{w}{g} A_c L_c\right) V^2 + (w A_t Y) \frac{Y}{2} = \frac{1}{2}\left(\frac{w}{g} A_c L_c\right) V^2_0 \qquad (28)$$

or

$$\alpha V^2 + \beta Y^2 = \alpha V^2_0 \qquad (29)$$

in which α and β are constants.

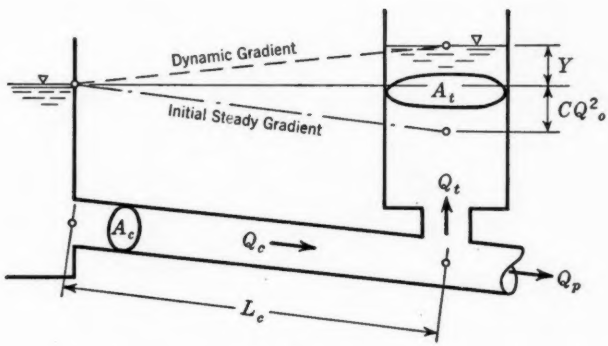


FIG. 7.—SURGE TANK ELEMENTS

This expression is therefore the equation of an ellipse which can be always transformed into a circle by the process of normalization that is, by making horizontal and vertical scales homogeneous. Thus, dividing through by the term (αV^2_0),

$$\frac{\alpha V^2}{\alpha V^2_0} + \frac{\beta Y^2}{\alpha V^2_0} = 1 \qquad (30)$$

$$\left(\frac{V}{V_0}\right)^2 + \left(\frac{Y}{Y_0}\right)^2 = 1 \qquad (31)$$

in which

$$Y_0 = V_0 \sqrt{\frac{A_c L_c}{A_t g}} = V_0 Z_0 \qquad (32)$$

with $Z_0 = \sqrt{\frac{A_c L_c}{A_t g}}$ defined as the surge impedance of the tank. This situation is sketched in Fig. 8(a), in which it is seen that a maximum positive surge occurs at point 2, for which

$$Y_{max} = Y_0 \qquad (33)$$

or

$$Y_{max} = V_0 \sqrt{\frac{A_c L_c}{A_t g}} = \text{"free" surge} \qquad (34)$$

From Fig. 8(a) it is also clear that the tank will oscillate indefinitely as a result of continuous interchange of energy without dissipation.

In the practical case, however, conduit friction will modify the basic energy expression into the form:

$$\text{PE} = \text{KE} + (\text{work done against friction}) \quad (35)$$

Thus the total change of water level in the tank will be increased by the presence of friction, although the surges will dampen out successively as a result of dissipation, as shown in Fig. 8(b).

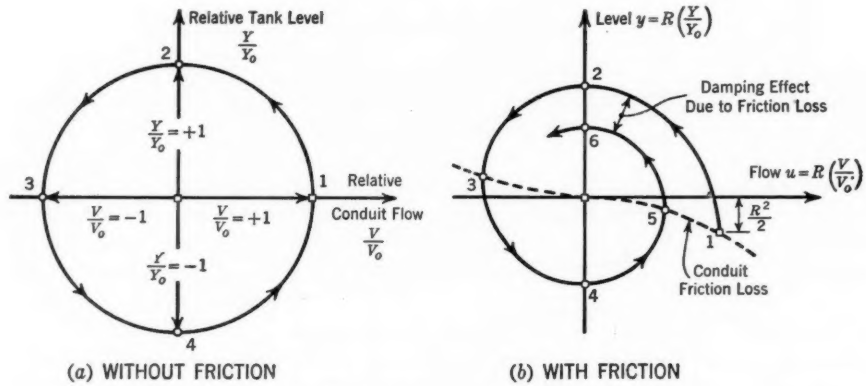


FIG. 8.—SURGE ENERGY DIAGRAMS

The effect of friction, for any given transient, can be measured if the ratio of friction loss to free surge is known. Thus the parameter may be defined

$$R = \frac{2\,H_f}{Y_0} = \frac{2\,H_f}{V_0}\sqrt{\frac{A_t\,g}{A_c\,L_c}} \quad (36)$$

in which H_f is the rated conduit friction loss. For a particular value of R the surge oscillations would be modified as shown. It should be noted that the initial friction loss relative to Y_0 is given by $R/2$.

It has been found useful and convenient to multiply both the horizontal (conduit flow $= u$) and vertical (tank level $= y$) scales by the constant factor R, to obtain the normalized plots shown in Fig. 9.

Further analysis will show that in terms of the notation used in the diagram—

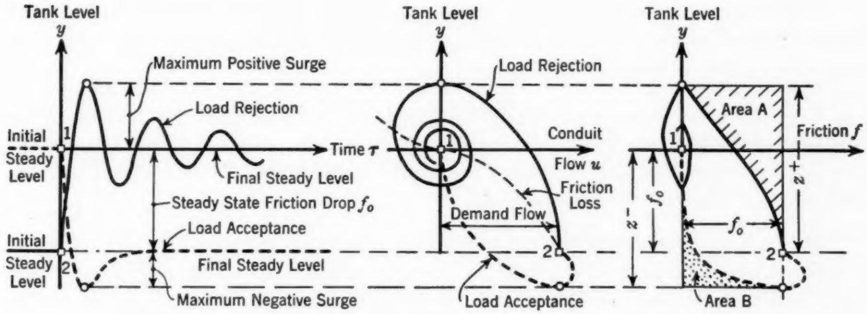
$$\text{PE} = \text{KE} + (\text{work done}) \quad (37)$$

becomes—

$$\frac{z^2}{2} = \frac{u^2}{2} + \int f\,dz \quad (38)$$

Eq. 38 resolves the problem of computing surges into the evaluation of the work integral on the right-hand side, corresponding to the areas A and B in Fig. 9. This can be done by several different procedures and has been successfully achieved using the electronic computer.

The value of the particular formulation of the problem as implied by Eqs. 37 and 38 lies in the considerable precision that may be derived using even approximate methods of integration. Employing such techniques, tables of surges that are accurate to four significant figures have been prepared for the normal range of tank design. Similar procedures have been developed for restricted orifice and differential tanks.



(a) SURGE-TIME CURVE (b) SURGE-FLOW CURVE (c) SURGE-FRICTION CURVE

FIG. 9.—SURGE TANK TRANSIENTS

It is also of interest to note that the tank constant R serves as a dimensionless hydraulic similitude index similar to the Reynolds, Froude, and Mach numbers. In other words, two simple surge tanks A and B will be dynamically similar if

$$R_a = R_b \qquad (39)$$

or

$$\frac{2\,H_{fa}}{V_a\sqrt{\dfrac{A_{ca}\,L_{ca}}{A_{ta}\,g}}} = \frac{2\,H_{fb}}{V_b\sqrt{\dfrac{A_{cb}\,L_{cb}}{A_{tb}\,g}}} \qquad (40)$$

This conclusion agrees with the tank similitude analyses of A. H. Gibson[35,36] and W. F. Durand.[37]

SURGE TANK PROBLEMS

The excellent early studies of R. D. Johnson[5] were essentially limited only to establishing design constants and dimensions for fixed positive and negative surge heights. Mr. Johnson did not attempt the determination of the various relationships between diverse values of the many system constants, but rather established working design values for a particular choice of these constants; these restrictions appreciably simplified the analysis of performance, leading directly to the type of results that Mr. Johnson sought.

[35] "The Investigation of the Surge Tank Problem by Model Experiments," by A. H. Gibson, *Proceedings*, Inst. C. E., London, England, 1924–1925.

[36] "A Comparison of Observations on Surge Tank Installations and on Their Scale Models," by A. H. Gibson, *ibid.*, 1932–1933.

[37] "Application of the Law of Kinematic Similitude of the Surge-Chamber," by W. F. Durand, *Transactions*, ASME, Vol. 43, 1921, p. 1177.

However, many tanks now installed are often required to operate under conditions quite different from those selected for design, and the behavior under these conditions has sometimes been unfavorable and unexpected. The problem of determining the response of tanks under all reasonable disturbances that might be encountered has been neglected in the past, largely because of the altogether forbidding length of time required by even the most elementary investigations using conventional methods.

With these needs in view, the research efforts reported in this paper have been devoted, in the surge tank field, to the following problems concerning present and proposed tank installations:

1. The behavior of tank systems for operating conditions other than those used for design; and
2. The true stability margins for surge tanks subjected to specified disturbances of appreciable magnitude.

It has long been realized that the actual transient behavior of surge tanks is considerably influenced by the action of the turbine governor and other plant and load characteristics. However, the magnitude of this effect depends on the

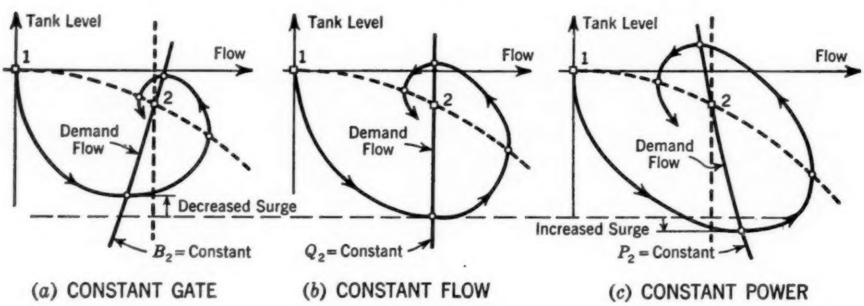


FIG. 10.—DEMAND FLOW ASSUMPTIONS

comparative values of the governor timing and the natural period of the tank. Investigations by the writer have established the limitations and range of validity of three simplifying assumptions as to plant effects, as illustrated in Fig. 10—constant gate opening, constant discharge, and constant input power.

1. Constant Gate Opening (B_2 = Constant) at the Final Steady-State Value.—This is the situation most representative of a unit under manual control by the operator, and produces the least surge, under otherwise comparable conditions, as a result of increased positive damping action.

2. Constant Discharge (Q_2 = Constant) at the Final Steady-State Value.—This was the basis of the methods of Mr. Johnson and most other investigators and produces a greater surge than assumption 1.

3. Constant Input Power (P_2 = Constant) to the Runner at the Final Steady-State Value.—This assumes that efficiency changes are small and the turbine regulates over a time that is very short compared to the tank period. This

assumption gives materially greater surges, and can even lead to instability, as explained subsequently in Fig. 14.

This method, when suitably linearized, was the basis of the classical tank stability analyses[7] that have been modified by later work[38,39,13] of J. Calame and D. Gaden, G. Evangelisti, and the writer.[39]

In addition to the conventional problem of a sudden or step-change in demand conditions, the electronic computer has been applied successfully in the examination of two other types of problems—synchronous or resonant load changes and true static and dynamic stability margins.

4. Synchronous or Resonant Load Changes, of the Pulsing and Oscillatory Type.—Such a loading program may result in abnormally high surges and pressure swings as the period of the disturbance approaches the natural period of the tank. Previously, this problem had been investigated only for a handful of cases.[40]

5. True Static and Dynamic Stability Margins for Tanks Operating Both Under Steady-Load Conditions and Under Appreciable Changes in Demand.—The simplified techniques of the linear stability theory, for example, cannot account for the known augmentation to stability through the differential principle, giving rather the same value for the critical tank area for all common types of tank. Moreover, the engineering concept of stability implies far more than the various classical definitions of mathematics and mechanics. If a surge tank is to be stable in a real sense, it must be able to reach and maintain an effective steady operating condition in a sufficiently short time after any reasonable load disturbance. Simply having the oscillations die out in a time just short of eternity is not adequately meeting these requirements, yet quantitative, generally applicable knowledge has been lacking in these matters.

With the conclusion of a first stage in the current research program at M.I.T., it is possible to give a few preliminary examples of the many results obtained.

Simple Tank Equations

For computational purposes, it is usually convenient to return to the basic differential equations in the physical form; for the simple tank, following the nomenclature of Fig. 6(*b*),

Continuity—

$$- A_t \frac{dY}{dt} = Q_p - Q_c \text{.....................} (41)$$

Acceleration—

$$-\frac{Lc}{A_c\, g} \frac{dQ_c}{dt} = Y + C\, Q^2{}_c \text{....................} (42)$$

Assuming a uniform conduit and tank, with square law friction, these may be

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[38] "De la stabilité des instalations hydrauliques munies des chambres d'equilibre," by J. Calame and D. Gaden, *Schweizerische Bauzeitung*, Band 90, 1927.

[39] "The Stability of Surge Tanks," by H. M. Paynter, thesis presented to Massachusetts Institute of Technology, at Cambridge, Mass., in 1949, in partial fulfilment of the requirements for the degree of Master of Science.

[40] "The Differential Surge Tank," by R. D. Johnson, *Transactions*, ASCE, Vol. 78, 1915, p. 760.

normalized to the form:

Continuity—

$$-\frac{dy}{d\tau} = v - u \qquad (43)$$

Acceleration—

$$-\frac{du}{d\tau} = y + \tfrac{1}{2} u^2 \qquad (44)$$

in which $y = R\,(Y/Y_0)$; $u = R\,(Q_c/Q_0)$; $\tau = 2\,\pi\,t/T_{ct}$; $v = R\,(Q_p/Q_0)$; $R = 2\,H_f/Y_0$; and $T_{ct} = 2\,\pi\sqrt{\frac{A_t\,L_c}{A_c\,g}}$ = free period. As a particular case one

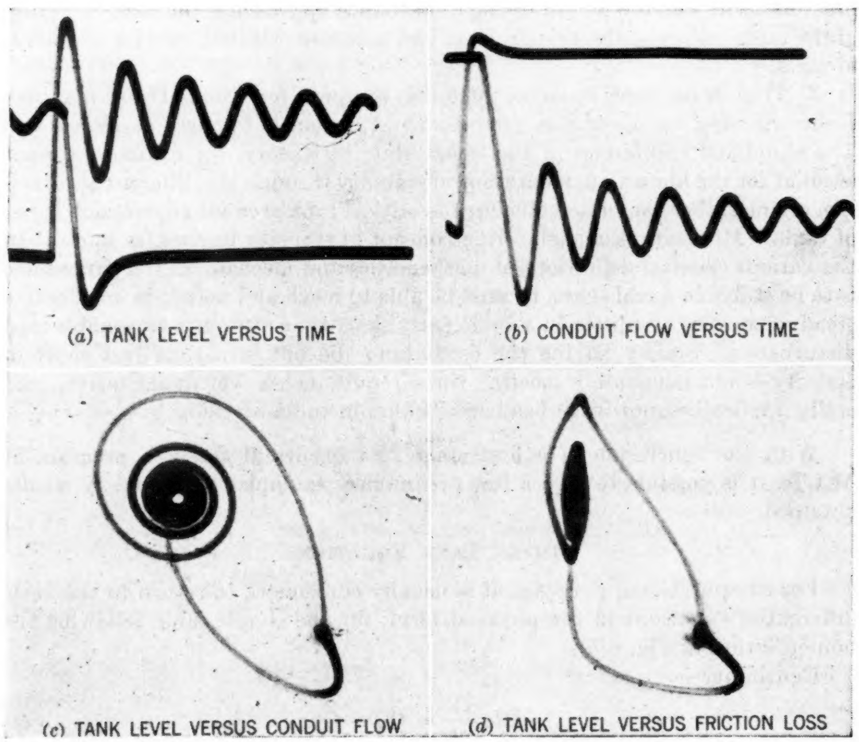

FIG. 11.—REPRESENTATIVE COMPUTER SOLUTIONS OF SURGE TANK TRANSIENTS

might take the transient from an initial steady state $v_1 = R_1 = 1.7$ to a final steady state $v_2 = R_2 = 0$ that might be either full or partial rejection to zero for a great number of practical instances. This transient is indicated in Figs. 9 and 11.

Thus, any given plant can be specified by the value of the parameter R for rated conditions, and the particular transients are specified by the initial and final steady states $u_1 = v_1 = R_1$ and $u_2 = v_2 = R_2$, respectively.

The work of W. E. Milne in this field[41,42,43] involved a complete investigation by numerical methods of the system of Eqs. 43 and 44. Mr. Milne also described the general case of a quadratically damped vibration; but his solutions were only for a step change in demand flow, assumption 2. Nevertheless, his tables use the identical normalized variables as outlined in this paper and it is hoped that this paper may serve as another introduction to Mr. Milne's valuable work.

Nonlinear Features

The two principal nonlinearities of the simple surge tank equations, as outlined, lie in the demand flow characteristic v and the friction loss term $u^2/2$.

1. The Demand Flow Characteristic v.—This factor may have any of the three forms indicated in Fig. 10, and may vary with time as well because of the finite governor time.

2. The Friction Loss Term $1/2\, u^2$.—This term has been assumed to be quadratic since analysis is somewhat simpler for this case; but in general an exponent different from 2 may have to be taken into account. Although numerical or graphical solutions of particular instances may be desirable, it has been found possible, by means of the electronic analog computer, to calculate complete and universal solutions of the surge tank equations for all cases normally arising in practice.

Computer Setup

The electronic computer used at the M.I.T. Hydrodynamics Laboratory for research in the regulation field is of the high-speed analog type, and is ideally suited to this type of analysis. The basic computing epoch is four milliseconds and the system variables are represented by analogous voltages functionally related through appropriate computer components that solve the fundamental equations.[44,45,46]

The general arrangement of the computer is illustrated in Fig. 12 along with operational details of representative banks of components. In particular, Fig. 12(d) shows the arrangement of components and the block diagram for simple tank transients following sudden changes in demand flow. It is of especial interest to note the $(C - \sigma - C)$-bank and its behavior as outlined in Fig. 12(c); it is this bank that represents the square-law friction.

Typical computer solutions, as photographed from the oscilloscope screen, are shown in Fig. 11. These may be compared to the solution diagrammed in Fig. 9. All problems may be set up in such a way that calibration and scaling can be made directly from the final photographic records. This arrangement

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[41] "Damped Vibrations; General Theory Together with Solutions of Important Special Cases," by W. E. Milne, Univ. of Oregon Publications, Eugene, Ore., August, 1923.

[42] "Tables of Damped Vibrations," by W. E. Milne, Univ. of Oregon Publications, Eugene, Ore., March, 1929.

[43] "Tables of Derivatives for Damped Vibrations," by W. E. Milne, Oregon State College Monographs, Corvallis, Ore., December, 1935.

[44] "The Study of Oscillatory Circuits by Analog Computer Methods," by H. Chang, R. C. Lathrop, and J. C. Rideout, *Proceedings*, National Electronic Conference, 1950.

[45] "Catalog and Manual," G. A. Philbrick Researches Inc., Boston, Mass., 1950.

[46] "The Electronic Analog Computer as a Laboratory Tool," by G. A. Philbrick and H. M. Paynter, *Industrial Laboratories*, May, 1952.

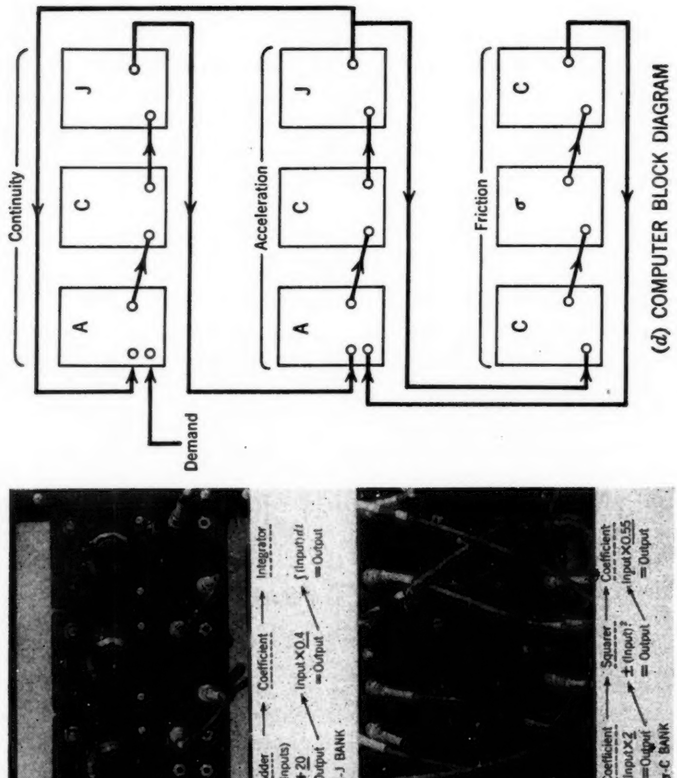

FIG. 12.—ARRANGEMENT OF COMPUTER

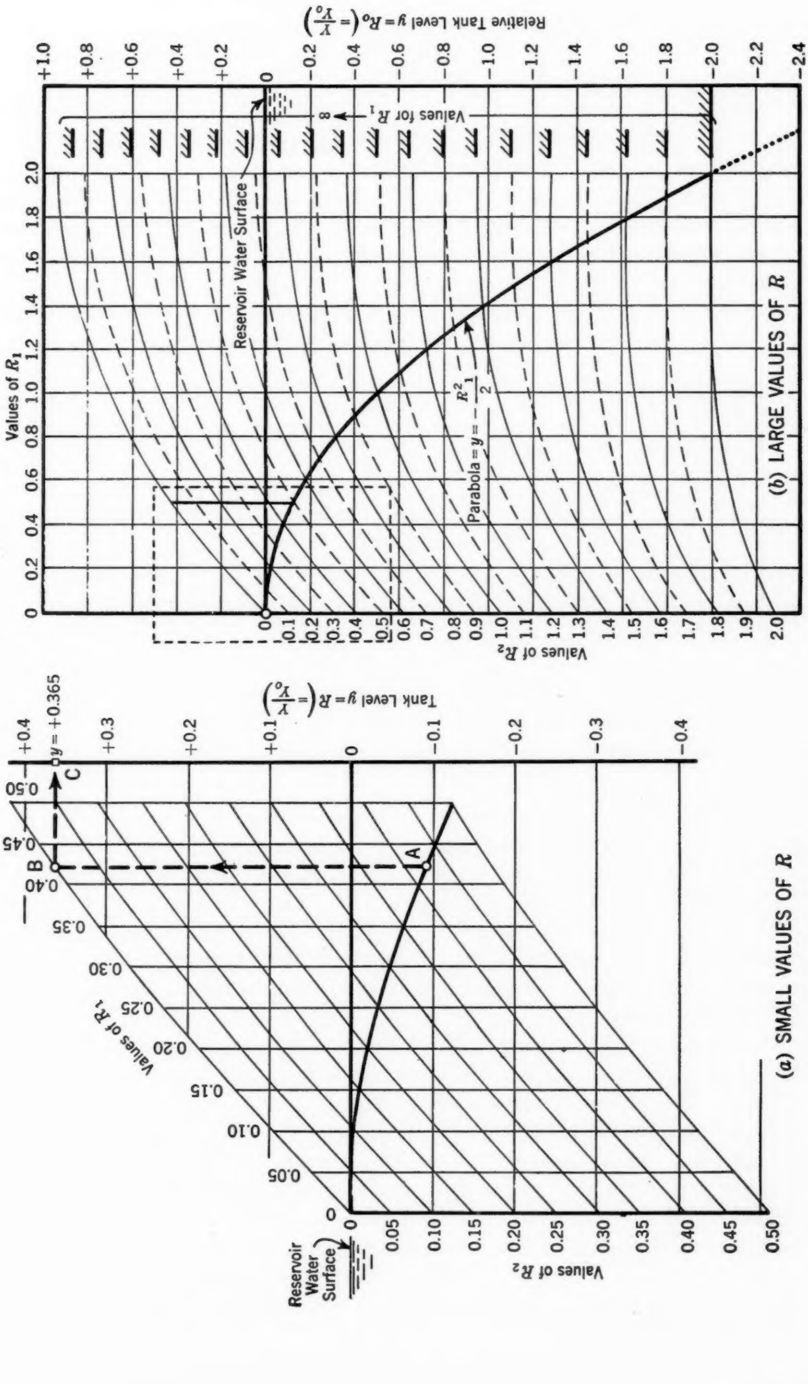

FIG. 13.—SIMPLE TANK SURGE CURVES

has been found very successful in practice and constitutes another strong argument for the normalization procedures that make it possible.

Analysis and Results

It is worthwhile to present two examples of the many charts that have been assembled from the computer studies on simple surge tanks. Similar curves have been obtained for other types of tanks.

The first specimen, shown in Fig. 13, is a family of curves, plotted similar to the method indicated by Mr. Durand,[37] from which the maximum or minimum tank level resulting from any load acceptance or rejection transient with any simple tank may be computed, subject to the assumptions of constant demand flow and square-law friction. This plot may be compared with the earlier Johnson plot.[47] An example of its use is given in the Appendix.

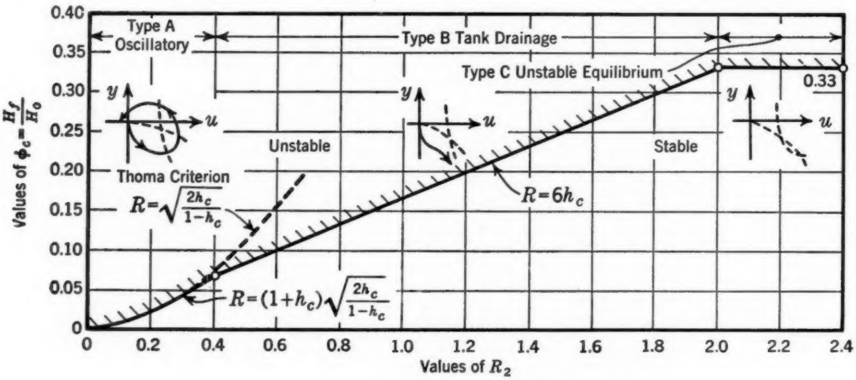

Fig. 14.—Surge Tank Stability

The second curve, Fig. 14, has to do with surge tank stability for full-load acceptance and compares the conditions of the classical criterion[7] with actual computed solutions assuming constant input power, plotting the required minimum value of the simple, tank constant R for any given value of conduit friction loss relative to static head, $h_c \equiv H_{f0}/H_0$. The curve is seen to be composed of three sections corresponding to the three types of instability indicated. There are, in order—the oscillatory region, the drainage region, and the region of unstable steady state.

1. Oscillatory Region (Small Frictional Effect).—In this range stable response (below the curve) is indicated by damped oscillations and unstable behavior is evidenced by oscillations of growing magnitude.

2. Drainage Region (Medium Frictional Effect).—For this region satisfactory behavior results in rapidly damped return to full-load gradient conditions. However, if the tank is of insufficient size, the conduit flow would fail to accelerate as fast as the tank was being drained, and, in the practical case, the unit would evidence instability by failure to deliver the required power at the bottom of the swing. This might result in loss of synchronism.

[47] "Hydroelectric Handbook," by W. P. Creager and J. D. Justin, John Wiley & Sons, Inc., New York, N. Y., 1950, Chapter XXV.

3. Region of Unstable Steady State (High Frictional Effect).—When $h_c > 1/3$, the static equilibrium is unstable and the slightest disturbance would result in unstable response. Stating the fact another way, if the rated friction loss H_{f0} is greater than one third the static head H_0, stable regulation is impossible regardless of the presence or size of the tank.

Comments.—Of course, all these conclusions are based on the assumption of constant power input to the prime mover (case 3). However, the effect of the finite time taken by the governor to respond to the changes of head as well as the effects of interconnected generating units usually are such as to ameliorate these conditions. Nevertheless, the curves serve the useful purpose of design limitations and it should be noted that the conventional allowance for drooping efficiency curves applied to the Thoma criterion may still be inadequate for the larger values of h_c.

The use of these curves is shown in the Appendix for an actual surge tank installation in which field tests were made to check the design assumptions.[48]

Conclusion

It is hoped that this outline of the type of methods used in the research program at M.I.T. will stimulate new interest in the desirability of obtaining more complete and effective solutions of problems in the regulation field. Preliminary results have given ample evidence that much valuable information can be derived through use of appropriate analogies, whereby results in other fields (for instance, electrical engineering) may be profitably interpreted in an hydraulic context.

In addition, it is believed that adaptation of electronic computers to surge and water hammer studies offers new possibilities for obtaining results of general and permanent usefulness to problems whose solution is impractical by any other means.

Acknowledgment

The work described in this paper was made possible by a grant from the Research Corporation of New York, N. Y. The computer components are of a stock commercial type as manufactured by George A. Philbrick Researches, Inc., of Boston, Mass.

APPENDIX. TYPICAL EXAMPLES

Explanatory Notes.—The following two examples have been included primarily to demonstrate the use and limitations of the design curves presented in the paper. In particular, the second example, treating surges in the simple surge tank at Tallulah Falls, Ga., was chosen from among the many field test results principally because the deviations from computed studies (obtained by electronic computer or any other means) represent the typical range of uncertainty encountered in practice.

[48] "Tests Check Computed Values of Surges," by Eugene Lauchli, *Engineering Record*, Vol. 71, 1915, p. 378.

It is not intended that the very brief exposition of methods presented be considered complete in itself, but it is outlined only to demonstrate the value of generalized curves and formulas for rapid approximate computations.

The material in Example 1 is drawn from the curves presented in Figs. 3 and 4. A complete explanation and theoretical treatment of water hammer resonance have been presented by Mr. Gaden[8] and Mr. Evangelisti.[13] The confirmation of these results by a graphical solution follows well-known methods.[16,17,18]

However, in Example 2, in addition to the use of the surge diagram of Fig. 13, certain approximate formulas for times and alternate surges have been employed that were not described in the text of the paper. These rough formulas have been found to give good results when checked against both the writer's theoretical studies and field tests, and are similar to certain expressions proposed in the work of Mr. Milne.[41] An explanation of the nature and use of diagrams similar to Fig. 13 is given in the paper of Mr. Durand.[37]

Example 1. Water Hammer Resonance.—A hypothetical hydroelectric plant with the following physical constants is to be analyzed:

Factor	Value
Penstock length L, in feet	1288
Rated head H_0, in feet	200
Rated velocity V_0, in feet per second	10
Wave velocity a, in feet per second	3220

From these data the elastic constants K and μ may be obtained. Thus,

$$K = \frac{c\,V_0}{g\,H_0} = \frac{3220 \times 10}{32.2 \times 200} = 5 \qquad (45)$$

$$\mu = \frac{2\,L}{c} = \frac{2 \times 1288}{3220} = 0.8 \text{ sec} \qquad (46)$$

If it is assumed that gate oscillations exist with a period of 4.8 sec and a swing of 20% range, the problem is concerned with determining the resultant head swings in the penstock, where the natural frequency is $T_n = 2\,\mu = 1.6$ sec. Thus, the data, concerning gate oscillations, required for using the resonance diagram are: The oscillations are assumed sinusoidal about a 100% gate opening, in a range of $\pm 10\%$. The relative period $T_0/T_n = 4.8/1.6 = 3$.

Referring to the resonance diagram (Fig. 4), $\phi_h = 161°$ and $h/b = 1.90$. The head oscillations are thus found to range over $(1.90 \times 20 =)$ 38%, the period for this case being $3\,T_n = 6\,\mu = 4.8$ sec, with a phase lag ϕ of 161°.

The foregoing determination can be checked graphically by reference to Fig. 15. Thus, for a gate swing of 20% and a period (as before) of $T_0 = 3\,T_n = 6\,\mu = 4.8$ sec, the head range is found to be 38% with a phase lag of 165°.

Although the resonance diagrams presented were derived on the basis of small changes, the agreement with conventional computations for large changes is satisfactory for most purposes. However, the upward shift in the head variation curve should be noted, since this assymmetry may reach substantial proportions in particular cases.

Example 2. Surge Tank at Tallulah Falls.[48]—The design data and general computations in Table 2 are self-explanatory. The use of these preliminary data is demonstrated for three cases.

Case A, $R_1 = 0.425$ and $R_2 = 0$.—Enter Fig. 13(a) at $R_1 = 0.425$ (point A), read vertically upward to intersect at $R_2 = 0$ (point B). From this point, read

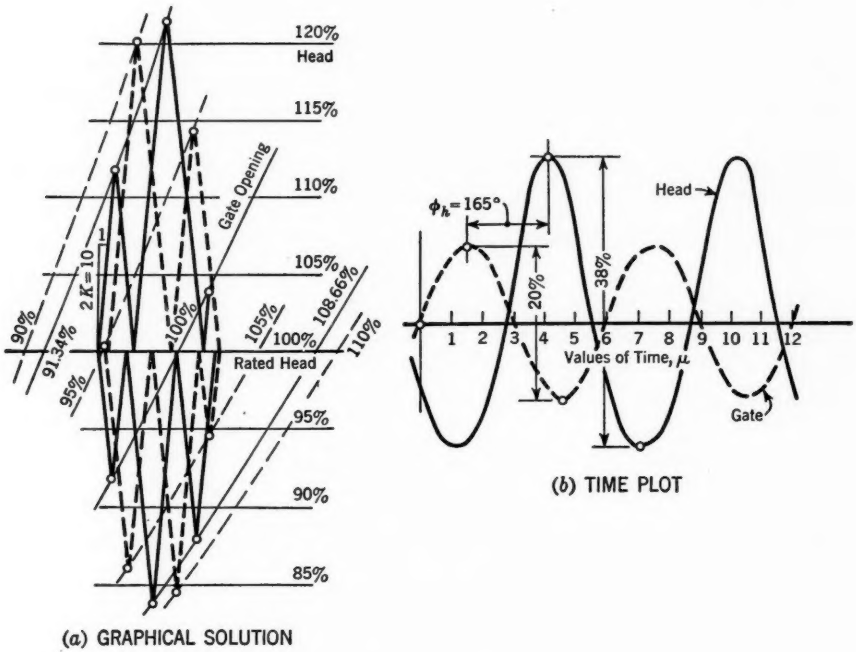

FIG. 15.—EXAMPLE OF RESONANCE

horizontally to point C on the scale at the right, obtaining $y_A = +0.365$. The initial friction drop may be evaluated as $f_A = \frac{1}{2} R^2{}_1 = 0.091$, or may be read directly from the chart at point A.

Thus, in summary,

Relative surge y_A, in feet above the reservoir surface	+0.365
Relative friction drop f_A, in feet below reservoir surface	0.091
Relative level change from the initial level $z_A = y_A + f_A$	0.456

These values can be converted to physical magnitudes most easily through the proportional relationship:

$$\frac{\text{Level change (in feet)}}{\text{Friction drop (in feet)}} = \frac{\text{relative change } (z)}{\text{relative friction } (f)} \qquad (47)$$

TABLE 2.—DESIGN OF SURGE TANK AT TALLULAH FALLS, GA. (EXAMPLE 2)

Case	DESIGN DATA[a]		OBSERVED DATA			COMPUTATIONS		
	Velocity, ΔV (ft per sec)	Friction loss (ft)	Velocity increment (ft per sec)	Level change (ft)	Time to maximum change (sec)	Free surges[b] $Y_0 = \Delta V \times Z_0$ (ft)	Transients[c] $R = \frac{2 H_f}{Y_0}$	
							R_1	R_2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	6.13	5.0	6.13→0	23.6	120	6.13×3.83 = 23.5	2×5/23.5 = 0.425	0
B	3.18	1.5	3.18→0	11.9	80	3.18×3.83 = 12.2	0.425×3.18/6.13 = 0.222	0
C	0.16	(0.2)	0.16→1.6	−6.65	100	−1.44×3.83 = −5.52	0.425×0.16/6.13 = 0.011	..[d]

[a] The tank in this example has a rectangular section, 30 ft by 71 ft, with a cross section area of (A_t = 30 × 71 =) 2131 sq ft. The length L_c of the conduit is 6666 ft and its area A_c is 151 sq ft. The observed friction measurements are given in Cols. 1 and 2.

[b] Surge impedance Z_0 (Eqs. 32 and 34) $= \sqrt{\frac{L_c A_c}{g A_t}} = \sqrt{\frac{6666 \times 151}{32.2 \times 2130}} = 3.83$ sec

Free period $T_0 = 2\pi \sqrt{\frac{L_c A_t}{g A_c}} = 2\pi \sqrt{\frac{6666 \times 2130}{32.2 \times 151}} = 342$ sec.

[c] See Eq. 36. [d] In case C, R_2 (Col. 8) = 0.425 × 1.6/6.13 = 0.111, the same as R_1.

or

$$\text{Level change (in feet)} = \left(\frac{z}{f}\right) \times \text{friction drop (in feet)} \quad \ldots\ldots (48)$$

Thus, level change $= \left(\frac{0.456}{0.091}\right) \times 5.0 = 25.0$ ft; and time to maximum—

$$T_m = \frac{T_0}{4}\left(1 + \frac{2 R_1}{\pi}\right) = 85\left(1 + \frac{0.85}{3.14}\right) = 108 \text{ sec}$$

alternate surges for corresponding surge numbers (i):

$$\left|\frac{1}{y_{i+1}}\right| = \left|\frac{1}{y_i}\right| + \frac{2}{3};\ T_{i+1} = T_m + i\frac{T_0}{2} = 108 + i\,170.$$

The computation of surge time for i = 1 to 5 is listed in Table 3(a).

Case B. $R_1 = 0.222$ *and* $R_2 = 0$.—Enter Fig. 13(a) at $R_1 = 0.222$, read vertically upward to intersect at $R_2 = 0$. From this point, read horizontally to the scale at the right, obtaining $y_B = +0.207$. As in case A, the initial friction drop may be evaluated as $f_B = \frac{1}{2} R^2_1 = 0.022$, or may be read directly from Fig. 13(a). In other words,

Relative surge y_B, in feet above the reservoir surface	+0.207
Relative friction drop f_B, in feet below reservoir surface	0.022
Relative level change from the initial level	0.229

Again, applying the relationships expressed by Eqs. 47 and 48,

TABLE 3.—COMPUTATIONS OF SURGE TIME

Surge number i	(a) CASE A				(b) CASE B				(c) CASE C		
	y_i	$\frac{1}{y_i}$	Surge (ft)	Time (sec)	y_i	$\frac{1}{y_i}$	Surge (ft)	Time (sec)	y_i	Surge (ft)	Time (sec)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	0.365	2.740 +0.667	+20.0	108 +170	0.207	4.831 +0.667	+11.0	97 +170	−0.100	−5.5	92
2	0.293	3.407 +0.667	−16.1	278 +170	0.182	5.498 +0.667	− 9.7	267 +170	+0.069	+3.8	262
3	0.245	4.074 +0.666	+13.4	448 +170	0.162	6.165 +0.666	+ 8.6	437 +170	−0.073	−4.0	432
4	0.211	4.740 +0.667	−11.6	618 +170	0.146	6.831 +0.667	− 7.8	607 +170	+0.047	+2.6	602
5	0.185	5.407	+10.1	788	0.134	7.498	+ 7.1	777	...	...	...

Time to first maximum—

$$T_m = \frac{T_0}{4}\left(1 + \frac{2\,R_1 + R_2}{\pi}\right) = 85\left(1 + \frac{0.444}{3.14}\right) = 97 \text{ sec}$$

Alternate surges—

$$(\text{Amplitude}) \quad \frac{1}{y_{i+1}} = \frac{1}{y_i} + \frac{2}{3}$$

$$(\text{Time}) \quad T_{i+1} = T_m + i\,\frac{T_0}{2} = 97 + 170\,i.$$

The remaining steps in the computation of surge time are entered in Table 3(*b*).

Case C. $R_1 = 0.011$ *and* $R_2 = 0.111$.—Following the same procedure as in cases A and B,

Relative surge y_c, in feet above the reservoir surface	−0.100
Relative friction drop f_C, in feet below reservoir surface	0.000
Relative level change from the initial level	−0.100

Eqs. 47 and 48 yield:

Time to first maximum—

$$T_m = \frac{T_0}{4}\left[1 + \frac{2\,(R_1 + R_2)}{\pi}\right] = 85\left[1 + \frac{0.244}{3.14}\right] = 92 \text{ sec.}$$

Alternate Surges (amplitudes as in Fig. 16)—

$$(\text{Time}) \quad T_{i+1} = 92 + 170\,i.$$

The computations for the remainder of case C are shown in Table 3(*c*).

Plots.—The values of surge heights for the three cases given are plotted in Fig. 16, for comparison with the actual test records plotted as the smooth curves. The divergences for the load rejection cases are caused primarily by the slow rate at which the demand flow was reduced. On the other hand, for

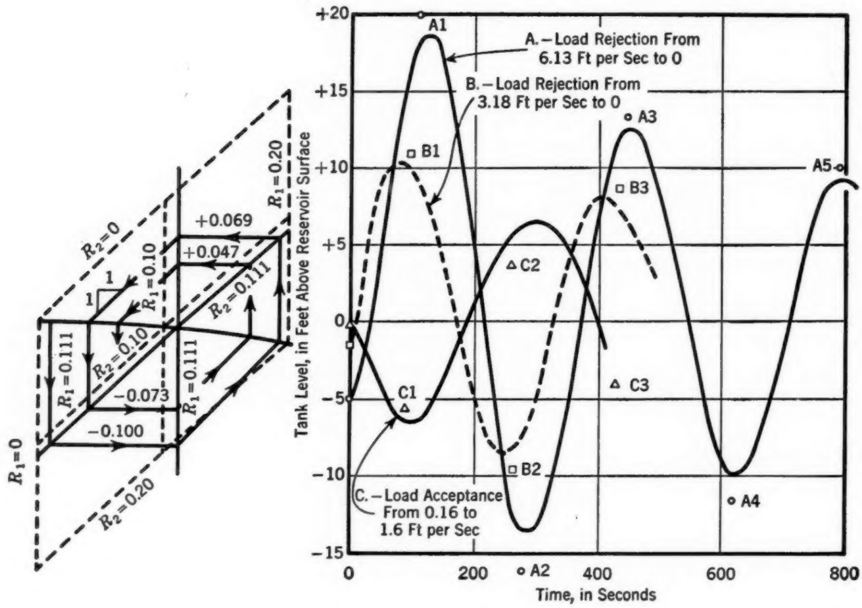

FIG. 16.—TRANSIENTS OF TALLULAH (GA.) TANK

the load-acceptance case, the actual curves may be expected to be in excess of the constant demand-flow results because of the effort of the governor to preserve constant power.

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